GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC TASK 2

2025

Instructions:

• Working time – 90 minutes +5 minutes reading time

• Write using black pen.

• Calculators approved by NESA may be used.

• A reference sheet is provided.

• For section 1, use the multiple-choice answer sheet provided. Attempt questions 1 - 5.

• For Questions in Section II, show relevant mathematical reasoning and/or calculations. Use the answer booklet provided. Attempt questions 6 – 11.

● Marks may be deducted for careless or badly arranged work.

Section 1 (5 marks)

Questions 1 – 5

Use the multiple-choice answer sheet for questions 1 to 5

1. Given $\underset{\sim}{v} = 6\underset{\sim}{i} - 7\underset{\sim}{j}$ and $\underset{\sim}{w} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$ find $2\underset{\sim}{v} - 6\underset{\sim}{w}$.

(A) $24\underset{\sim}{i} - 44\underset{\sim}{j}$

(B) $\underset{\sim}{i} - 44\underset{\sim}{j}$

(C) $24\underset{\sim}{i} + 16\underset{\sim}{j}$

(D) $24\underset{\sim}{i} - 16\underset{\sim}{j}$

2. A vector perpendicular to $3\underset{\sim}{i} + 4\underset{\sim}{j}$ and with a magnitude of 5 is:

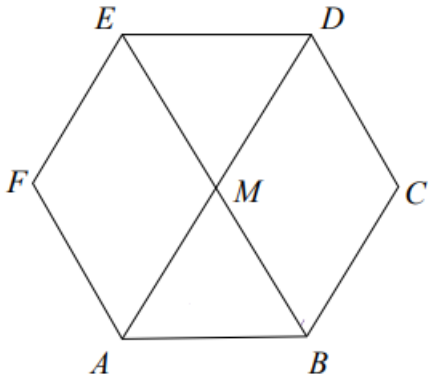
(A) $-4\underset{\sim}{i} - 3\underset{\sim}{j}$

(B) $5(4\underset{\sim}{i} - 3\underset{\sim}{j})$

(C) $4\underset{\sim}{i} - 3\underset{\sim}{j}$

(D) $25(4\underset{\sim}{i} - 3\underset{\sim}{j})$

3. In the diagram below, $ABCDEF$ is a regular hexagon with centre M .Choose the incorrect statement.



(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = \underset{\sim}{0}$

(B) $6\overrightarrow{AB} = \underset{\sim}{0}$

(C) $\overrightarrow{AD} + 2\overrightarrow{EF} = \underset{\sim}{0}$

(D) $\overrightarrow{BC} + \overrightarrow{EF} = \underset{\sim}{0}$

4. A vector $\underset{\sim}{u}$ has a magnitude of 10 units and direction of $-\frac{\pi}{3}$ from the positive $x-$axis. Which of the following represent the unit vector in the direction of $\underset{\sim}{u}$?

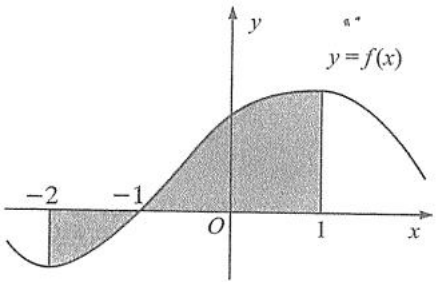
(A) $5\underset{\sim}{i}-5\sqrt{3}\underset{\sim}{j}$

(B) $\frac{\sqrt{3}}{2}\underset{\sim}{i}-\frac{1}{2}\underset{\sim}{j}$

(C) $5\sqrt{3}\underset{\sim}{i}-5\underset{\sim}{j}$

(D) $\frac{1}{2}\underset{\sim}{i}-\frac{\sqrt{3}}{2}\underset{\sim}{j}$

5. In the diagram shown, the total area of the two regions shaded is equal to:



(A) $\int_{1}^{-1} f(x)dx-\int_{-2}^{-1} f(x)dx$

(B) $\int_{-1}^{1} f(x)dx+\int_{-1}^{-2} f(x)dx$

(C) $\int_{-1}^{1} f(x)dx+\int_{-2}^{-1} f(x)dx$

(D) $\int_{-1}^{1} f(x)dx-\int_{-2}^{-1} f(x)dx$

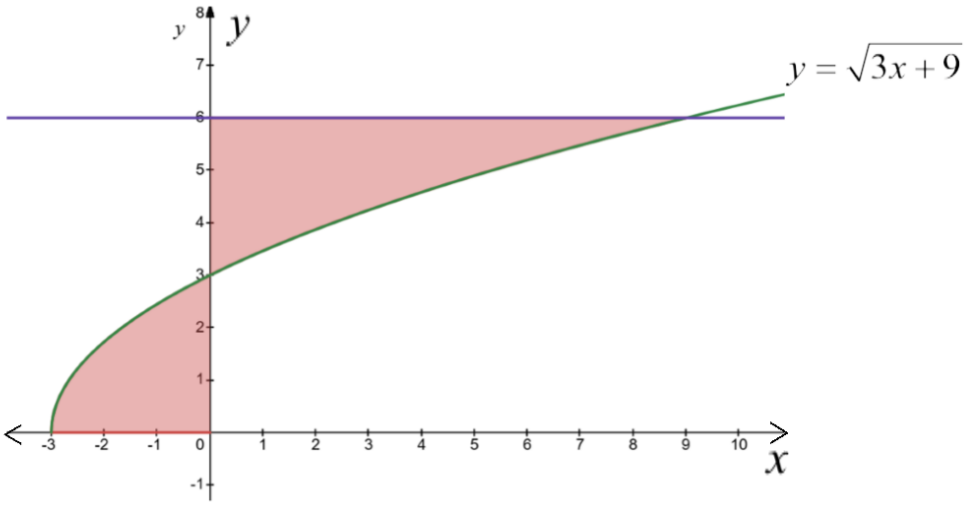
Section II (82 marks)

● Answer questions in the answer booklet provided.

● Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (14 marks)

(a) Calculate the shaded area. **4**



(b) (i) Show that $\dfrac{5x+1}{x+1} = 5 - \dfrac{4}{x+1}$ **1**

(ii) Hence find $\int \dfrac{5x+1}{x+1}\,dx$. **3**

(c) (i) Find $\dfrac{d}{dx}\left(x\log_e x\right)$ **2**

(ii) Hence find $\int \log_e x\,dx$ **2**

(iii) Find the area bounded by the graph of $y = \log_e x$, the $x-$axis, $x=1$ and $x=4$ giving exact answers. **2**

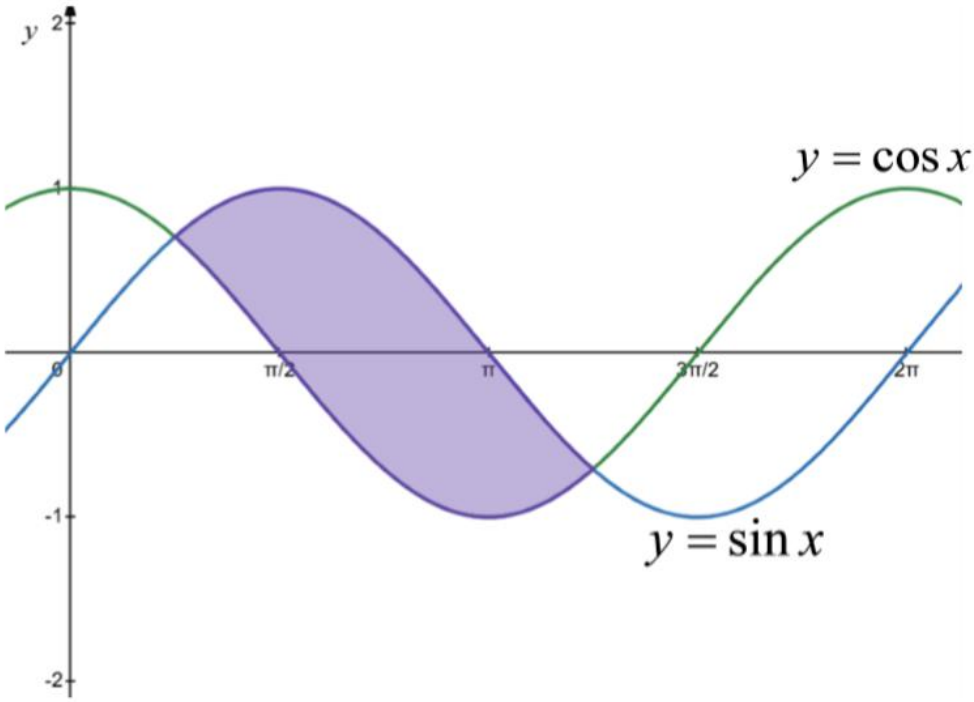
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Question 7 (10 marks)

(a) Find the equation of the normal to the curve $y = \ln\left(\dfrac{2x-1}{x+1}\right)$, at the point where $x = 2$. **3**

(b) Find $f(x)$ if $f'(x) = 3\cos\dfrac{x}{4}$ and $f(\pi) = 9\sqrt{2}$. **3**

(c) The graphs of $y = \sin x$ and $y = \cos x$ are drawn below.



(i) Find where the curves $y = \sin x$ and $y = \cos x$ intersect in the interval $0 \le x \le 2\pi$. **2**

(ii) Hence find the exact area of the shaded region. **2**

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Question 8 (14 marks)

(a) Given $\underset{\sim}{a} = -2\underset{\sim}{i} - 4\underset{\sim}{j}$, $\underset{\sim}{b} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$. Find:

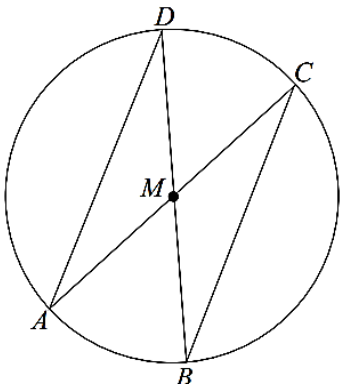
(i) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(ii) the exact angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ correct to the nearest degree. **2**

(b) Find the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ where $\underset{\sim}{a} = 10\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 7\underset{\sim}{j}$. **2**

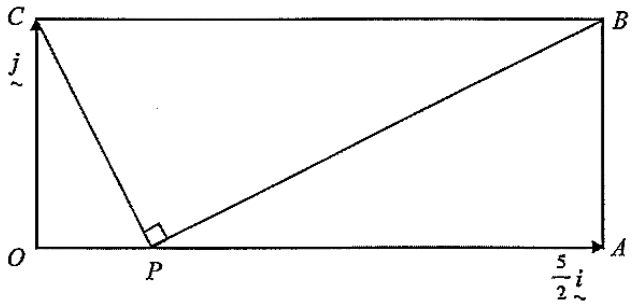
(c) In the diagram below, AC and BD are two diameters of the circle with centre M.

Using vector addition, show that $\overrightarrow{AD} + \overrightarrow{DB} + \overrightarrow{BC} + \overrightarrow{CA} = \underset{\sim}{0}$ **2**



(d) In the diagram, $OABC$ is a rectangle in which $\overrightarrow{OA} = \frac{5}{2}\underset{\sim}{i}$ and $\overrightarrow{OC} = \underset{\sim}{j}$.

P is a point on OA such that $\overrightarrow{OP} = \lambda\underset{\sim}{i}$ for some parameter λ and $\angle CPB = 90°$.



(i) Write $\overrightarrow{PC}$ and $\overrightarrow{PB}$ in component form or as a column vector. **2**

(ii) Use vector methods to show that $2\lambda^2 - 5\lambda + 2 = 0$. **3**

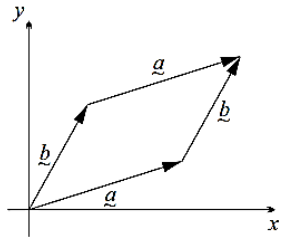
(iii) Hence find any values of λ. **2**

Please turn over

Question 9 (13 marks)

(a) Given $\underset{\sim}{a} = -2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 7\underset{\sim}{j}$, find a scalar p such that $\underset{\sim}{a} + p\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$. **3**

(b) Consider the parallelogram formed by the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ as shown in the diagram.



(i) Write down vectors representing the diagonals of the parallelogram in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

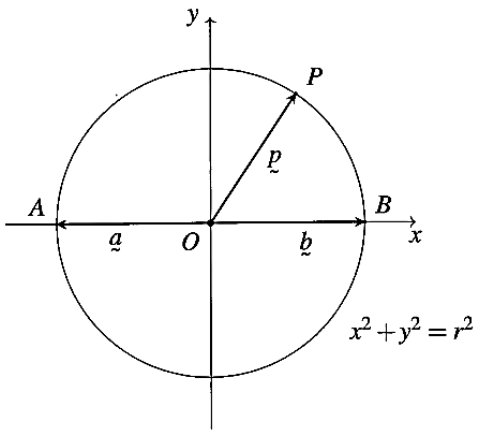
(ii) Using vectors, show that the diagonals of the parallelogram are perpendicular when $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ **2**

(c) The vectors $\underset{\sim}{p} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{q} = -5\underset{\sim}{i} + n\underset{\sim}{j}$ are parallel. Find the value of n. **2**

(d) The point P with position vector $\underset{\sim}{p} = p_1\underset{\sim}{i} + p_2\underset{\sim}{j}$ lies on the circle $x^2 + y^2 = r^2$.

Points A and B are the x-intercepts of the circle with position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, respectively.

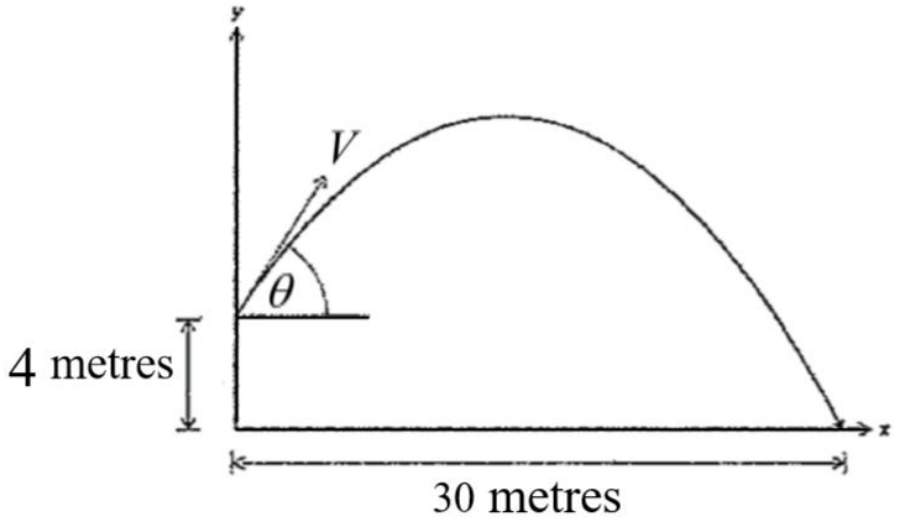
Show that $\left|\underset{\sim}{p} - \underset{\sim}{a}\right|^2 + \left|\underset{\sim}{p} - \underset{\sim}{b}\right|^2 = 4r^2$ **4**



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Question 10 (11 marks)

(a) A rock is projected with a speed of $V\ m/s$ from a point 4 metres above a flat ground. The angle of projection to the horizontal is θ as shown.



(i) Taking the ground as the origin and the acceleration due to gravity as $10m/s^2$,

Show that $x = Vt\cos\theta$ and $y = Vt\sin\theta - 5t^2 + 4$ **4**

(ii) Show that the cartesian equation of the path is given by

$$y = x\tan\theta - \frac{5x^2}{V^2}\left(1+\tan^2\theta\right)+4$$ **2**

(iii) If the rock hits the ground 30 metres away, find the value of V correct to 2 decimal places if $\theta = \tan^{-1}\left(\frac{5}{12}\right)$. **3**

(iv) Find the maximum height reached correct to 2 decimal places. **2**

Please turn over

Question 11 (15 marks)

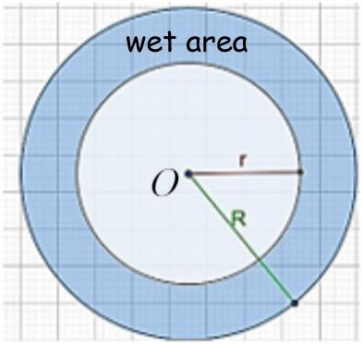
(a) (i) Prove that the horizontal range of a projectile with velocity $V\ m/s$ and angle of projection α is given by $\dfrac{V^2 \sin 2\alpha}{g}$ metres where g m/s^2 is the acceleration due to gravity. The position of the particle at any time t is given by

$$x = Vt\cos\alpha,\ y = Vt\sin\alpha - \frac{1}{2}gt^2 \qquad \textbf{(Do not prove this)}$$ **3**

(ii) Find the angle α for the range to be maximised. **2**

(iii) Find the maximum range. **1**

(iv) A garden sprinkler sprays water symmetrically about its vertical axis at a constant speed of $V\ m/s$. As the jet of water rotate through $360°$, the angle of the jet varies from $15°$ to $60°$ to the horizontal. From a fixed point O on the level ground, the sprinkler will wet the surface in an annular region with centre O as shown in the diagram. Show that the area of the annular region is $\dfrac{3\pi V^4}{4g^2}$ square metres. **3**



(b) A particle is projected to hit an object which is on top of a fence 2 metres high, and 40 metres away. The initial velocity is $30m/s$. The position of the particle at any time t is given by $x = (30\cos\alpha)t,\ y = (30\sin\alpha)t - 5t^2$ **(Do not prove this)**

(i) Show that $40\tan^2\alpha - 180\tan\alpha + 49 = 0$ where α is the angle of projection. **3**

(ii) Hence find the two angles of projection. **3**

End of Examination

Please remember to write 'End of Solutions' on your answer paper.

12 Extension 1 Task 2, 2025 – Solutions

Multiple choice (5 marks)

1. $2\underset{\sim}{v} - 6\underset{\sim}{w}$

$= 2(6\underset{\sim}{i} - 7\underset{\sim}{j}) - 6(-2\underset{\sim}{i} + 5\underset{\sim}{j})$

$= 12\underset{\sim}{i} - 14\underset{\sim}{j} - (-12\underset{\sim}{i} + 30\underset{\sim}{j})$

$= 12\underset{\sim}{i} - 14\underset{\sim}{j} + 12\underset{\sim}{i} - 30\underset{\sim}{j}$

$= 24\underset{\sim}{i} - 44\underset{\sim}{j}$ (A)

2. $(3\underset{\sim}{i} + 4\underset{\sim}{j}) \cdot (4\underset{\sim}{i} - 3\underset{\sim}{j}) = 0$

$|4\underset{\sim}{i} - 3\underset{\sim}{j}| = \sqrt{4^2 + 9} = 5$

(C)

3. $6\overrightarrow{AB}$ is a vector in the direction of $\overrightarrow{AB}$. Its magnitude is 6 times the magnitude of $\overrightarrow{AB}$.

$6\overrightarrow{AB} \neq \underset{\sim}{0}$ (B)

4. $\underset{\sim}{u} = 10\cos\left(\frac{\pi}{3}\right)\underset{\sim}{i} + \left(-10\sin\frac{\pi}{3}\right)\underset{\sim}{j}$

$= 5\underset{\sim}{i} + (-5\sqrt{3})\underset{\sim}{j}$

Unit vector $= \frac{5}{10}\underset{\sim}{i} + \frac{-5\sqrt{3}}{10}\underset{\sim}{j}$

$= \frac{1}{2}\underset{\sim}{i} - \frac{\sqrt{3}}{2}\underset{\sim}{j}$ (D)

5) D or B

Marker's Comments:

Question 6 (14 marks)

(a) $y = \sqrt{3x+9}$

$y^2 = 3x+9$

$3x = y^2 - 9$

$x = \frac{y^2-9}{3}$

shaded Area

$= \int_3^6 \frac{y^2-9}{3}\,dy - \int_0^3 \frac{y^2-9}{3}\,dy$

I_1 $\qquad$ I_2

$I_1 = \int_3^6 \frac{y^2-9}{3}\,dy$

$= \frac{1}{3}\left[\frac{y^3}{3} - 9y\right]_3^6$

$= \frac{1}{3}\left\{\left(\frac{216}{3} - 54\right) - \left(\frac{27}{3} - 27\right)\right\}$

$= \frac{1}{3}\left(72 - 54 - 9 + 27\right)$

$= \frac{1}{3} \times 36 = 12$

$I_2 = \int_0^3 \frac{y^2-9}{3}\,dy$

$= \frac{1}{3}\left[\frac{y^3}{3} - 9y\right]_0^3$

$= \frac{1}{3}\left\{\left(\frac{27}{3} - 27\right) - 0\right\}$

$= \frac{1}{3}(9 - 27) = \frac{-18}{3} = -6$

Area $= I_1 - I_2$

$= 12 - (-6)$

$= 18$ square units. (4)

Marker's comments:

Q6(b)(i)

$$RHS = 5 - \frac{4}{x+1}$$

$$= \frac{5x+5-4}{x+1}$$

$$= \frac{5x+1}{x+1}$$

$$= LHS.$$

(1)

(ii) $$\int \frac{5x+1}{x+1}\,dx$$

$$= \int \left(5 - \frac{4}{x+1}\right) dx$$ (3)

$$= 5x - 4\log_e(x+1) + C$$

(iii) $$\int_1^4 \log_e x\,dx$$

$$= \left[x\log_e x - x\right]_1^4$$

$$= (4\log_e 4 - 4) - (0-1)$$

$$= 4\log_e 4 - 4 + 1$$

$$= 4\log_e 4 - 3$$ (2)

Marker's Comments:

(c)(i) $$\frac{d}{dx}(x\log_e x) = x \times \frac{1}{x} +$$

$$= x \times \frac{1}{x} + \log_e x \times 1$$ (2)

$$= 1 + \log_e x$$

$$\int (1 + \log_e x)\,dx = x\log_e x + C$$

$$\int dx + \int \log_e x\,dx = x\log_e x + C$$ (2)

$$\int \log_e x\,dx = x\log_e x - x + C$$

Question 7 (10 marks)

(a) when $x=2$, $y=\ln\left(\frac{3}{3}\right)=0$

Point is $(2,0)$

$$y=\ln\left(\frac{2x-1}{x+1}\right)$$

$$=\ln(2x-1)-\ln(x+1)$$

$$y'=\frac{2}{2x-1}-\frac{1}{x+1}$$

when $x=2$, $y'=\frac{2}{3}-\frac{1}{3}$

$$=\frac{1}{3}$$

Gradient of the normal $=-3$

Equation of normal

$$y-0=-3(x-2)$$

$$y=-3x+6$$ (3)

(b) $f'(x)=3\cos\frac{x}{4}$

$$f(x)=12\sin\frac{x}{4}+C$$

$$9\sqrt{2}=12\times\sin\frac{\pi}{4}+C$$

$$9\sqrt{2}=12\times\frac{1}{\sqrt{2}}+C$$

$$C=9\sqrt{2}-\frac{12}{\sqrt{2}}=\frac{18-12}{\sqrt{2}}$$

$$=\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}$$

$$f(x)=12\sin\frac{x}{4}+3\sqrt{2}$$ (3)

(c)(i) $\sin x=\cos x$

$\tan x=1$

$x=\frac{\pi}{4}$ or $\pi+\frac{\pi}{4}$

$=\frac{\pi}{4}$ or $\frac{5\pi}{4}$ (2)

(ii) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,dx$

$$=\left[-\cos x-\sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$=\left(-\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4}\right)-\left(-\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\right)$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$$

$$=\frac{4}{\sqrt{2}}=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$ (2)

$$=2\sqrt{2}\text{ square units.}$$

(a) Mistake in y'
Some found equation of tangent.

(b) done well

(c) Mistakes in finding the solution of $\sin x=\cos x$

Question 8 (14 marks)

(a)(i) $\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= -8 - 12$

$= -20$ (1) well done

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$|\underset{\sim}{a}| = \sqrt{2^2 + 4^2}$

$= \sqrt{20}$

$|\underset{\sim}{b}| = \sqrt{4^2 + 3^2}$

$= 5$ (2)

$-20 = 5\sqrt{20} \cos\theta$

$\cos\theta = \frac{-20}{5\sqrt{20}} = \frac{-4}{\sqrt{20}} = \frac{-4}{2\sqrt{5}}$

$\theta = 153^\circ$ well done

(b) $\underset{\sim}{a} = 10\underset{\sim}{i} - 2\underset{\sim}{j}$,

$\underset{\sim}{b} = \underset{\sim}{i} - 7\underset{\sim}{j}$

$\hat{\underset{\sim}{b}} = \frac{\underset{\sim}{i} - 7\underset{\sim}{j}}{\sqrt{1+49}} = \frac{\underset{\sim}{i} - 7\underset{\sim}{j}}{\sqrt{50}}$

$\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}} = (10\underset{\sim}{i} - 2\underset{\sim}{j}) \cdot \frac{(\underset{\sim}{i} - 7\underset{\sim}{j})}{\sqrt{50}}$

$= (10\underset{\sim}{i} - 2\underset{\sim}{j}) \cdot \left(\frac{\underset{\sim}{i}}{\sqrt{50}} - \frac{7}{\sqrt{50}}\underset{\sim}{j}\right)$

$= 10 \times \frac{1}{\sqrt{50}} + \frac{-2 \times -7}{\sqrt{50}}$

$= \frac{10+14}{\sqrt{50}} = \frac{24}{\sqrt{50}}$

Vector projection $= (\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}}$

$= \frac{24}{\sqrt{\ }}\left(\frac{\underset{\sim}{i} - 7\underset{\sim}{j}}{\sqrt{50}}\right)$

$= \frac{24}{50}(\underset{\sim}{i} - 7\underset{\sim}{j})$ (2) well done

$= \frac{12}{25}(\underset{\sim}{i} - 7\underset{\sim}{j}) = \frac{12\underset{\sim}{i}}{25} - \frac{84\underset{\sim}{j}}{25}$

Marker's Comments:

b Alternate solution

$\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|^2}\underset{\sim}{b}$

$\frac{(10\underset{\sim}{i} - 2\underset{\sim}{j}) \cdot (\underset{\sim}{i} - 7\underset{\sim}{j})}{\sqrt{1^2+7^2}^2}(\underset{\sim}{i} - 7\underset{\sim}{j})$

$\frac{10 \times 1 + -2 \times -7}{50}(\underset{\sim}{i} - 7\underset{\sim}{j})$

$= \frac{24}{50}(\underset{\sim}{i} - 7\underset{\sim}{j})$

$= \frac{12}{25}(\underset{\sim}{i} - 7\underset{\sim}{j})$

(c) $\overrightarrow{AD} + \overrightarrow{DB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AC} + \overrightarrow{CA}$ (2)

$= \underset{\sim}{0}$ must show vector addition to get full marks

(d)(i) $\overrightarrow{PC} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} \lambda \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -\lambda \\ 1 \end{pmatrix}$ 1

$\overrightarrow{PB} = \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} - \begin{pmatrix} \lambda \\ 0 \end{pmatrix}$ (2)

$= \begin{pmatrix} \frac{5}{2} - \lambda \\ 1 \end{pmatrix}$ 1

(ii) $\overrightarrow{PC} \cdot \overrightarrow{PB} = 0$ 1

$\begin{pmatrix} -\lambda \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{5}{2} - \lambda \\ 1 \end{pmatrix} = 0$

$-\lambda\left(\frac{5}{2} - \lambda\right) + 1 = 0$ 1

$-\frac{5\lambda}{2} + \lambda^2 + 1 = 0$ (3)

$-5\lambda + 2\lambda^2 + 2 = 0$

$2\lambda^2 - 5\lambda + 2 = 0$ 1

$2\lambda^2 - 5\lambda + 2 = 0, \quad 0 < \lambda < \frac{5}{2}$

$2\lambda^2 - 4\lambda - \lambda + 2 = 0$

$2\lambda(\lambda - 2) - (\lambda - 2) = 0$

$(\lambda - 2)(2\lambda - 1) = 0$

$\lambda = 2$ or $\lambda = \frac{1}{2}$ (2)

well done

Marker's comments:

Question 9 (13 marks)

$\underset{\sim}{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\underset{\sim}{a} + p\underset{\sim}{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + p\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4p \\ 7p \end{pmatrix}$

$= \begin{pmatrix} -2+4p \\ 3+7p \end{pmatrix}$

$(\underset{\sim}{a} + p\underset{\sim}{b}) \perp \underset{\sim}{a}$

ie $(\underset{\sim}{a} + p\underset{\sim}{b}) \cdot \underset{\sim}{a} = 0$

$\begin{pmatrix} -2+4p \\ 3+7p \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = 0$

$-2(-2+4p) + 3(3+7p) = 0$

$4 - 8p + 9 + 21p = 0$

$13p = -13$

$p = -1$ (3)

(b)(i) Diagonals are $\underset{\sim}{a} + \underset{\sim}{b}$ and $\underset{\sim}{a} - \underset{\sim}{b}$ (2)

(ii) $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b})$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2$

$= 0$ when $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ (2)

$(\underset{\sim}{a} + \underset{\sim}{b}) \perp (\underset{\sim}{a} - \underset{\sim}{b})$ when $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

(c) $\underset{\sim}{p} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\underset{\sim}{q} = \begin{pmatrix} -5 \\ n \end{pmatrix}$

$\underset{\sim}{p} = \lambda \underset{\sim}{q}$

$\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \lambda \begin{pmatrix} -5 \\ n \end{pmatrix}$

$-2 = -5\lambda \Rightarrow \lambda = \frac{2}{5}$ (2)

$5 = n\lambda$

$n = \frac{5}{\lambda} = 5 \times \frac{5}{2} = 12.5$

Marker's Comments:

a) Some students said that $(\underset{\sim}{a} + p\underset{\sim}{b}) \cdot \underset{\sim}{a} = -1$ as if two lines are perpendicular.
but mostly done well.

b) students need to set out their working correctly

c) Well done.

(d) $\underset{\sim}{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} -r \\ 0 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} r \\ 0 \end{pmatrix}$

$$\underset{\sim}{p} - \underset{\sim}{a} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} - \begin{pmatrix} -r \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} p_1 + r \\ p_2 \end{pmatrix}$$

$$\underset{\sim}{p} - \underset{\sim}{b} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} - \begin{pmatrix} r \\ 0 \end{pmatrix} = \begin{pmatrix} p_1 - r \\ p_2 \end{pmatrix}$$

$$|\underset{\sim}{p} - \underset{\sim}{a}|^2 + |\underset{\sim}{p} - \underset{\sim}{b}|^2$$

$$= (\underset{\sim}{p} - \underset{\sim}{a}) \cdot (\underset{\sim}{p} - \underset{\sim}{a}) + (\underset{\sim}{p} - \underset{\sim}{b}) \cdot (\underset{\sim}{p} - \underset{\sim}{b}) \quad (\because |\underset{\sim}{a}|^2 = \underset{\sim}{a} \cdot \underset{\sim}{a})$$

$$= \begin{pmatrix} p_1 + r \\ p_2 \end{pmatrix} \cdot \begin{pmatrix} p_1 + r \\ p_2 \end{pmatrix} + \begin{pmatrix} p_1 - r \\ p_2 \end{pmatrix} \cdot \begin{pmatrix} p_1 - r \\ p_2 \end{pmatrix}$$

$$= (p_1 + r)^2 + p_2^2 + (p_1 - r)^2 + p_2^2$$

$$= p_1^2 + 2p_1 r + r^2 + p_2^2 + p_1^2 - 2p_1 r + r^2 + p_2^2$$

$$= 2p_1^2 + 2p_2^2 + 2r^2 \qquad (4)$$

$$= 2(p_1^2 + p_2^2) + 2r^2$$

$$= 2r^2 + 2r^2 \quad (\because p \text{ lies on the circle } p_1^2 + p_2^2 = r^2)$$

$$= 4r^2$$

Alternative method

(d) $\underset{\sim}{p} - \underset{\sim}{a} = \overrightarrow{AP}$

$\underset{\sim}{p} - \underset{\sim}{b} = \overrightarrow{BP}$

$\angle BPA = 90°$ (angle in a semicircle)

$\therefore \Delta BPA$ is right angled.

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2 = |\overrightarrow{AB}|^2$ (Pythagoras theorem)

$|\underset{\sim}{p} - \underset{\sim}{a}|^2 + |\underset{\sim}{p} - \underset{\sim}{b}|^2 = |\overrightarrow{AB}|^2$

$|\overrightarrow{AB}| = 2r$

$\therefore |\underset{\sim}{p} - \underset{\sim}{a}|^2 + |\underset{\sim}{p} - \underset{\sim}{b}|^2 = (2r)^2 = 4r^2$

Marker's Comments:

* Lot of students had trouble answering this question correctly.
* Alternate methods were used and were awarded full marks if correct reasoning was given.
* Students need to put the vector signs where necessary as the meaning of the statement changes otherwise.

Question 10 (13 marks)

$\ddot{x} = 0$ — (1) $\qquad \ddot{y} = -10$ — (2)

$\dot{x} = A$ — (3) $\qquad \dot{y} = -10t + B$ — (4)

When $t=0$, $\dot{x} = V\cos\theta$, $y = V\sin\theta$

$V\cos\theta = A \qquad V\sin\theta = 0 + B$

$\dot{x} = V\cos\theta$ — (5) $\qquad \dot{y} = -10t + V\sin\theta$ — (6)

$x = (V\cos\theta)t + C$ — (7) $\qquad y = -5t^2 + (V\sin\theta)t + D$ — (8)

When $t=0$, $x=0$, $y=4$

$0 = 0 + C \qquad 4 = 0 + 0 + D$

$\therefore C = 0 \qquad D = 4$

$x = (V\cos\theta)t$ (4 marks)

$y = -5t^2 + (V\sin\theta)t + 4$

$= Vt\cos\theta$ — (9) $\qquad = Vt\sin\theta - 5t^2 + 4$ — (10)

(ii) From (9) $\quad t = \dfrac{x}{V\cos\theta}$

Substitute in (10)

$$y = V \times \frac{x}{V\cos\theta}\sin\theta - \frac{5x^2}{V^2\cos^2\theta} + 4$$ (2 marks)

$$= x\tan\theta - \frac{5x^2}{V^2}\sec^2\theta + 4$$

$$= x\tan\theta - \frac{5x^2}{V^2}(1 + \tan^2\theta) + 4$$ — (11)

(iii) Substitute (30, 0) in (11)

$$0 = 30\tan\theta - \frac{5\times 30^2}{V^2}(1+\tan^2\theta) + 4$$

$$= 30\times\frac{5}{12} - \frac{5\times 900}{V^2}\left(1+\frac{25}{144}\right) + 4$$

$$0 = \frac{150}{12} - \frac{4500\times 169}{144V^2} + 4$$

$$\frac{4500\times 169}{144V^2} = \frac{198}{12}$$

$$V^2 = \frac{12\times 4500\times 169}{144\times 198}$$

3 marks

$$V = \underline{17.89 \text{ m/s}}$$

(iv) At the maximum height $\dot{y} = 0$

$$V\sin\theta - 10t = 0$$

$$10t = V\sin\theta$$

$$t = \frac{V\sin\theta}{10} = \frac{17.89\times\frac{5}{13}}{10} = 0.688 \text{ seconds}$$

Substitute in $y = Vt\sin\theta - 5t^2 + 4$

$$y = 17.89\times 0.688\times\frac{5}{13} - 5\times(0.688)^2 + 4$$

$$= \underline{6.37 \text{ metre}}$$

2 marks

Markers comments for Q10

(a) (i) Mostly done well.

(ii) Done well.

(iii) * Some students wrote when $x = 30$, $y = -4$. They didn't realise ground is the origin, i.e. when $x = 30$, $y = 0$

* Some students substituted the values (30, 0) correctly, but made calculation errors when finding the velocity.

iv) Mostly done well.

Question 11 (14 marks)

(a)(i) when the particle strikes the ground we must have $y = 0$

$$Vt\sin\alpha - \frac{1}{2}gt^2 = 0$$

$$t\left(V\sin\alpha - \frac{1}{2}gt\right) = 0$$

$$t = 0 \quad \text{or} \quad V\sin\alpha = \frac{1}{2}gt$$

$$t = 0 \quad \text{or} \quad t = \frac{2V\sin\alpha}{g}$$

Now $t = 0$ refers to the instant of projection and hence $t = \frac{2V\sin\alpha}{g}$ is the required time, the time of flight.

Substitute in $x = Vt\cos\alpha$

$$x = V\cos\alpha \times \frac{2V\sin\alpha}{g} = \frac{2V^2\sin\alpha\cos\alpha}{g}$$

$$R = \frac{V^2\sin 2\alpha}{g}$$

3

(ii) $R = \frac{V^2\sin 2\alpha}{g}$ is maximum when $\sin 2\alpha = 1$

ie when $\alpha = \frac{\pi}{4}$

2

Range is maximum when $\alpha = \frac{\pi}{4}$

and the maximum range is $\frac{V^2}{g}$

(iii) when $\alpha = 15°$, Range $= \frac{V^2 \sin 30}{g} = \frac{V^2}{2g}$

when $\alpha = 60°$, Range $= \frac{V^2 \sin 120}{g} = \frac{\sqrt{3}V^2}{2g}$

when $\alpha = 45°$, $R_{max} = \frac{V^2}{g}$

∴ the sprinkler will wet an annular region of internal and external radii $\frac{V^2}{2g}$ and $\frac{V^2}{g}$ respectively.

Area of wet surface

$$= \pi\left(\frac{V^2}{g}\right)^2 - \pi\left(\frac{V^2}{2g}\right)^2$$

$$= \frac{\pi V^4}{g^2} - \frac{\pi V^4}{4g^2}$$

$$= \frac{3\pi V^4}{4g^2}$$

3

(b) In order to hit the point (40, 2) we must have

$(30\cos\alpha)t = 40$ — (1)

$(30\sin\alpha)t - 5t^2 = 2$ — (2)

From ① $t = \frac{40}{30\cos\alpha} = \frac{4}{3\cos\alpha}$

Substitute in ②

$$(30\sin\alpha)\frac{\times 4}{3\cos\alpha} - \frac{5\times 16}{9\cos^2\alpha} = 2$$

$$40\tan\alpha - \frac{80}{9}\sec^2\alpha = 2$$

$$40\tan\alpha - \frac{80}{9}(1+\tan^2\alpha) = 2$$

$$360\tan\alpha - 80(1+\tan^2\alpha) = 18$$

$$360\tan\alpha - 80 - 80\tan^2\alpha = 18$$

$$80\tan^2\alpha - 360\tan\alpha + 98 = 0$$ (3)

$$40\tan^2\alpha - 180\tan\alpha + 49 = 0$$

$$\tan\alpha = \frac{180 \pm \sqrt{180^2 - 4\times 40\times 49}}{2\times 40}$$

$$= \frac{180 \pm \sqrt{24560}}{80} = 4.21 \text{ or } 0.29$$

$\alpha = \tan^{-1}(4.21)$ or $\tan^{-1}(0.29)$

$= 76^\circ 38'$ $= 16^\circ 10'$ (3)

There are two angles of projection to hit the point $(40, 2)$ which are $76^\circ 38'$ and $16^\circ 10'$

Marker's comments for Q11

(11)(a)(i) Generally very well done.

(ii) There were two choices – either use the maximum value of $\sin 2\alpha = 1$, so $2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$ or to use calculus to find the maximum value of $\frac{V^2 \sin 2\alpha}{g}$ [remembering that V & g are constants].

Generally well done.

(iii) Well done.

(iv) A disaster with a LOT of fudging.

Most students realised that the inner radius was when $\alpha = 15^\circ$ i.e. $\frac{V^2 \sin(2\times 15^\circ)}{g} = \frac{V^2 \sin 30^\circ}{g} = \frac{V^2}{2g}$

BUT they forgot that the OUTER radius would correspond to the MAXIMUM range ($\alpha = 45^\circ$)

so the OUTER radius $= \frac{V^2 \sin(2\times 45^\circ)}{g} = \frac{V^2 \sin 90^\circ}{g} = \frac{V^2}{g}$

MANY students either thought the outer radius when $\alpha = 60^\circ$ [which would give an outer radius of $\frac{V^2\sqrt{3}}{2g}$], which was NOT the outer radius & was worth NO marks.

The two most common FUDGES from here were for students to ignore the fact that the area is an ANNULUS & just do $\pi\left[\frac{V^2\sqrt{3}}{2g}\right]^2 = \frac{3\pi V^4}{4g^2}$ → This got NO marks as

(1) 60° was not the angle for EITHER the inner or outer radius
(2) It ignored the fact that the area was an ANNULUS.

OR for students to do $\pi\left(\left[\frac{V^2\sqrt{3}}{2g}\right]^2 - \left[\frac{V^2}{2g}\right]^2\right)$

$= \frac{3\pi V^4}{4g^2} - \frac{\pi V^4}{4g^2}$

and SOMEHOW arrive at $\frac{3\pi V^4}{4g^2}$ [obviously they think $3-1=3$].

This got 2 marks as (1) at least they got the INNER radius & (2) they realised the area was an ANNULUS

PTO →

Markers' comments for Q.(1) (continued).

Q.(1)(a)(iv) The point here is:

DON'T EVER FUDGE!

We will ALWAYS catch you!

[& so will the HSC markers].

(1)(b) Was generally very well done.